Shape Fluctuations and Shape Dynamics in Highly Excited Rotating Nuclei

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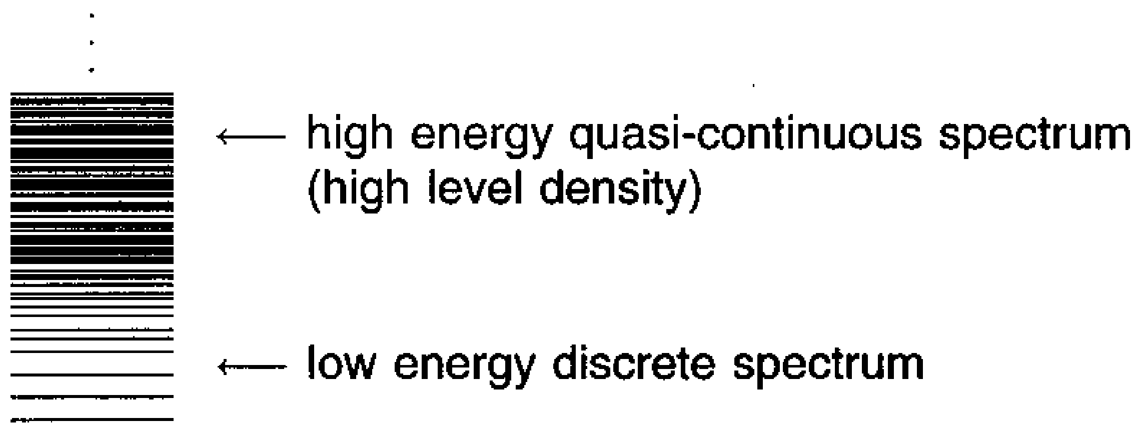
Outline

- introduction
- thermal ensembles of shapes
- physical observables in the adiabatic limit
  - giant dipole resonance
  - level density
- beyond the adiabatic limit: time-dependent fluctuations
- microscopic calculation of the diffusion coefficient for shape diffusion
- conclusions

What is a Hot Nucleus?

Nuclei formed in certain heavy ion reactions have high enough level density and large enough angular momentum to justify a statistical treatment.



Equilibrium assumption: at a given excitation energy E and spin J, all states have equal probability (microcanonical ensemble).

However, the nucleus is a finite system, so statistical methods must be applied with care.

Grand Canonical Ensemble in Nuclei

- relevant macroscopic variables:

extensive variable	intensive variable
energy E	temperature T
spin $\boldsymbol{J}$	angular velocity $\boldsymbol{\omega}$
particle number Z, N	chemical potential μ_{p}, μ_{n}

- representations related by Legendre transformation:

$$F(T, \boldsymbol{\omega}, \mu_{\mathrm{p}}, \mu_{\mathrm{n}}) = E - TS - \boldsymbol{\omega} \cdot \langle \hat{\boldsymbol{J}} \rangle - \mu_{\mathrm{p}} \langle \hat{Z} \rangle - \mu_{\mathrm{n}} \langle \hat{N} \rangle$$

  where F is the free energy and S the entropy

- expectations $\langle \cdot \rangle$ computed with the density matrix for the grand canonical ensemble

$$\rho = \exp\left[-\left(H - \boldsymbol{\omega} \cdot \boldsymbol{J} - \mu_{\mathrm{p}} Z - \mu_{\mathrm{n}} N\right) / T\right]$$

Level Density & Partition Function

- level density:

$$\rho(E, M, Z, N) = \mathrm{tr}\left[\delta(E - H)\delta_{M,\hat{J}_z}\delta_{Z,\hat{Z}}\delta_{N,\hat{N}}\right]$$

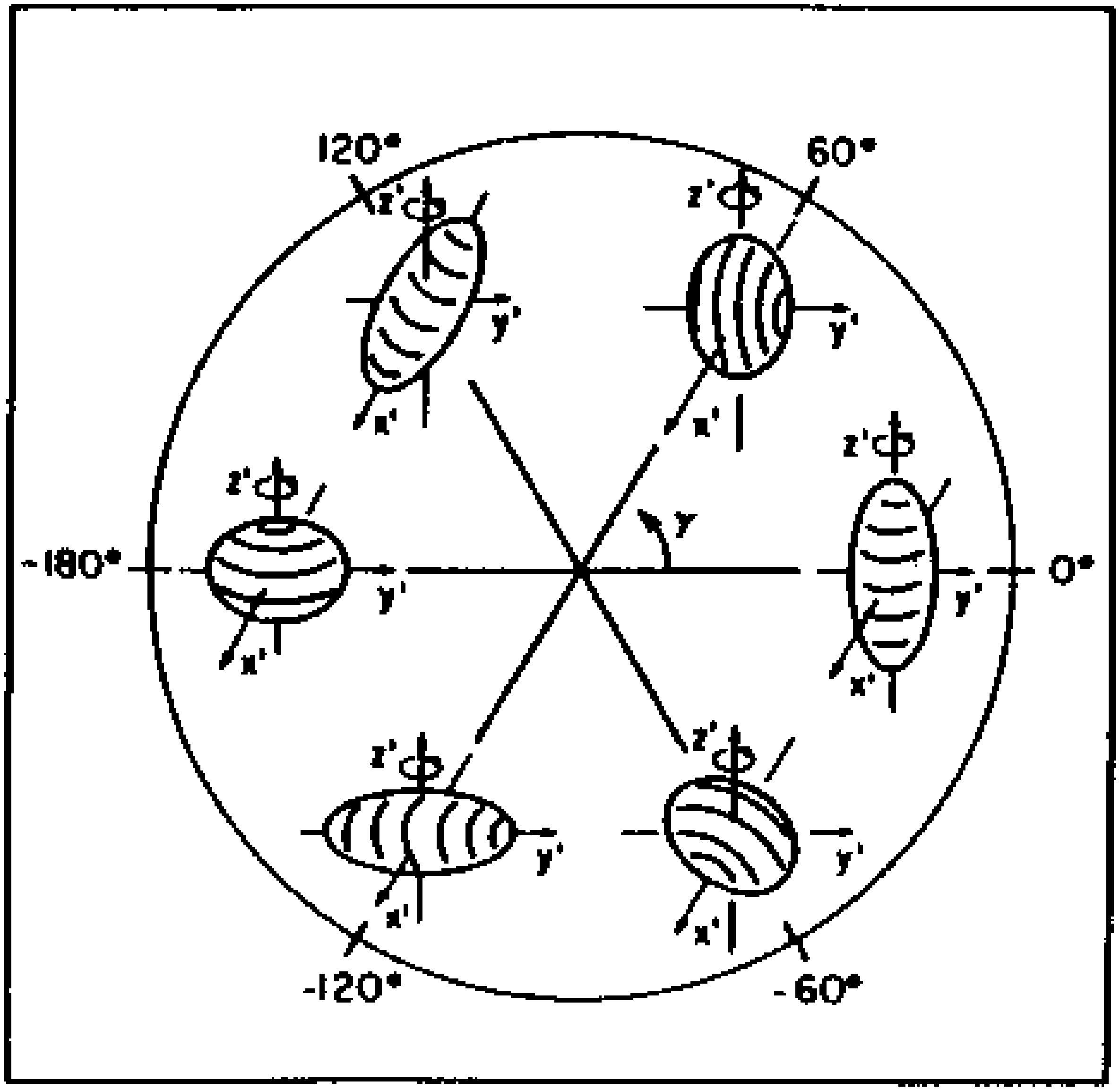
- partition function (Laplace transform):

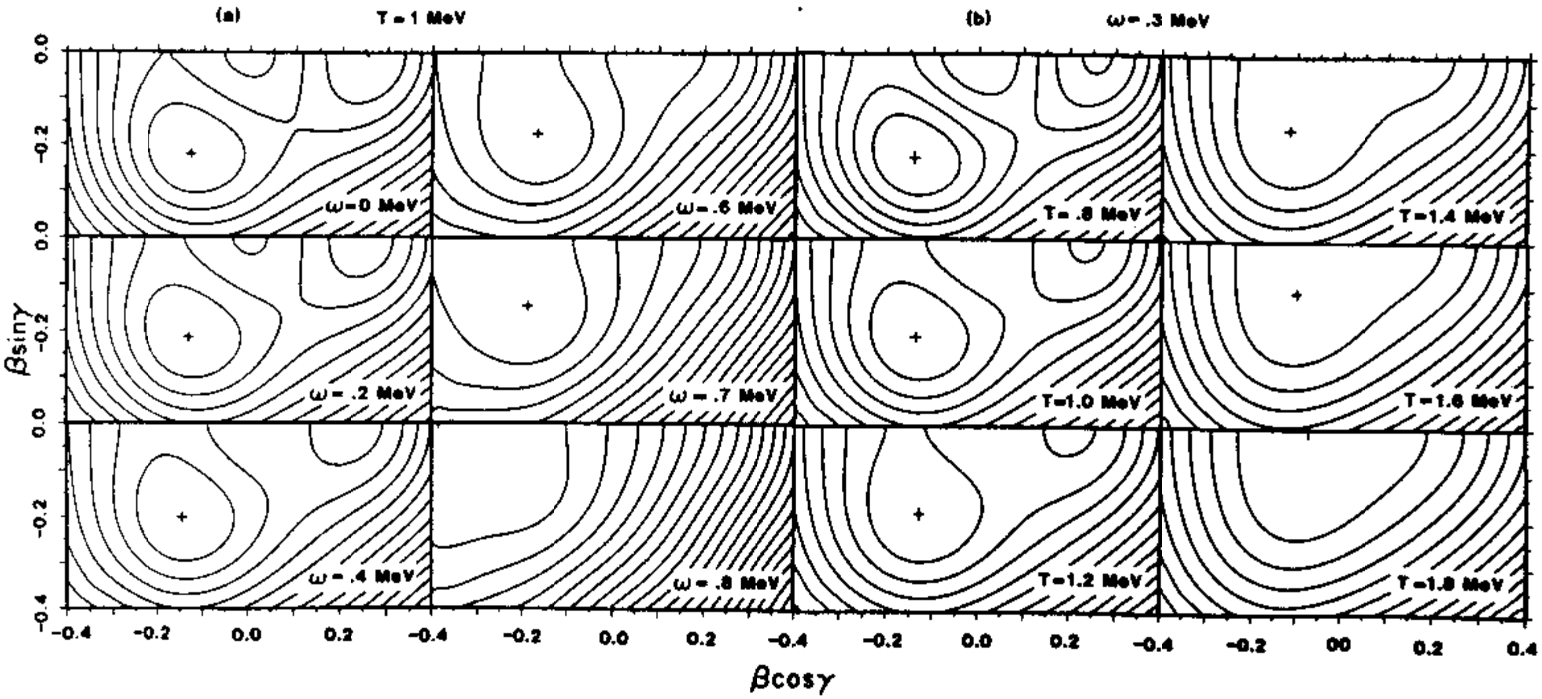
$$\begin{aligned}\mathcal{Z}(T, \omega_z, \mu_\mathrm{p}, \mu_\mathrm{n}) &= \int_0^\infty dE \sum_{M,Z,N} e^{-(E-\omega_z M-\mu_\mathrm{p} Z-\mu_\mathrm{n} N)/T} \rho(E, M, Z, N) \\ &= \mathrm{tr}\left\{\exp\left[-\left(H - \omega_z \hat{J}_z - \mu_\mathrm{p}\hat{Z} - \mu_\mathrm{n}\hat{N}\right)\Big/ T\right]\right\}\end{aligned}$$

- inverse relation (Mellin inversion):

$$\begin{aligned}\rho(E, M, Z, N) &= \frac{1}{32\pi^4} \int_{-i\infty}^{i\infty} d\left(\frac{1}{T}\right) \int_0^{4\pi i} d\left(\frac{\omega_z}{T}\right) \int_0^{2\pi i} d\left(\frac{\mu_\mathrm{p}}{T}\right) \int_0^{2\pi i} d\left(\frac{\mu_\mathrm{n}}{T}\right) \\ &\quad \times e^{(E-\omega_z M-\mu_\mathrm{p} Z-\mu_\mathrm{n} N)/T} \mathcal{Z}(T, \omega_z, \mu_\mathrm{p}, \mu_\mathrm{n})\end{aligned}$$

Quadrupole Shapes





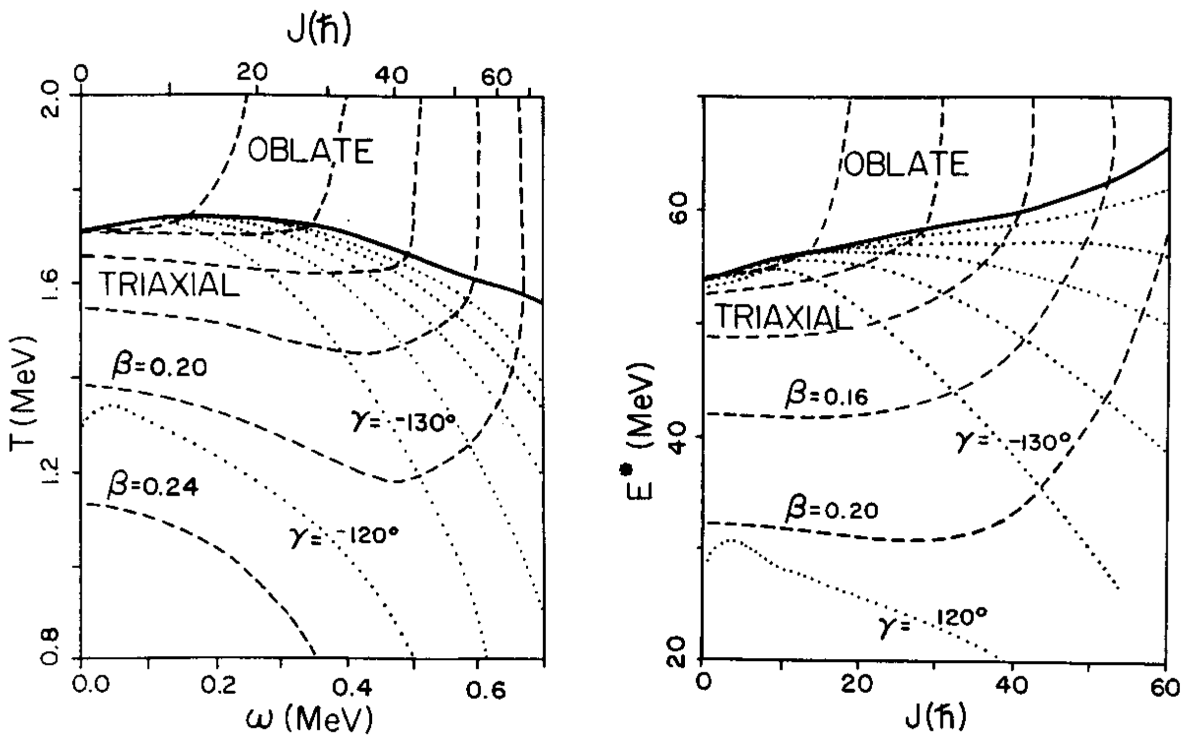
Typical free energy surfaces for ^{168}Er at (a) constant temperature T = 1 MeV; (b) at constant angular velocity ω = 0.3 MeV. Here $\boldsymbol{\omega}$ is parallel to the z' intrinsic axis. The contours are spaced 1 MeV apart. Notice the transition of the equilibrium shape (marked by "+") from prolate to oblate as (a) the angular velocity increases from 0 to 0.8 MeV or (b) as the temperature increases from T = 0.8 MeV to T = 1.8 MeV.

Landau Theory

- Free energy must be built from rotationally invariant combinations of $\alpha_{2\mu}$ and $\boldsymbol{\omega}$:

$$
\begin{aligned}
F(T,\boldsymbol{\omega},\alpha_{2\mu}) &= F(T,0,\alpha_{2\mu}) - \tfrac{1}{2}\boldsymbol{\omega}\cdot\mathbf{I}\cdot\boldsymbol{\omega} + \cdots \\
F(T,0,\alpha_{2\mu}) &= F_0 + A\beta^2 - B\beta^3\cos^3 3\gamma + C\beta^4 + \cdots \\
\mathsf{I}_{z'z'}(T,\alpha_{2\mu}) &= I_0 - 2R\beta\cos\gamma + 2I_1\beta^2 + 2D\beta^2\sin^2\gamma + \cdots
\end{aligned}
$$

- Deformation $\alpha_{2\mu}$ is the order parameter for a phase transition:
  - $\alpha = 0 \Rightarrow$ higher symmetry (spherical shape)
  - $\alpha \neq 0 \Rightarrow$ lower symmetry (deformed shape)
- Analysis of Landau expansion gives universal properties of shape transitions in nuclei.



Phase (shape) diagrams for ^{166}Er as calculated from a cranked Nilsson-Strutinsky model. Left: in intensive (T,ω) variables. Right: in extensive (E^*,J). The solid curve denotes the transition line separating the triaxial and oblate ($\gamma = -180°$) phases. Also shown are curves of constant β (dashed) and constant γ (dotted). In the upper part of the figures $\gamma = -180°$.

Nilsson Model

- Use cranked Nilsson Hamiltonian to find single-particle levels:

$$H = -\frac{\hbar^2\nabla^2}{2m} + \frac{m}{2}\sum_{i=1}^{3}\tilde{\omega}_i^2 {x_i}^2 - \kappa\hbar\tilde{\omega}_0\left[2\mathbf{l}\cdot\mathbf{s} + \mu\left(\mathbf{l}^2 - \langle\mathbf{l}^2\rangle_N\right)\right] - \boldsymbol{\omega}\cdot\mathbf{j}$$

  where

$$\begin{aligned}\tilde{\omega}_j &= \tilde{\omega}_0 \exp\left[-\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma - \frac{2\pi}{3}j\right)\right] \\ \hbar\tilde{\omega}_0 &= (41\ \mathrm{MeV})A^{-1/3}\left(1 \pm \frac{1}{3}\frac{N-Z}{A}\right)\end{aligned}$$

- Include Strutinsky correction to find free energy:

$$F(T,\omega,\mu,\beta,\gamma) = E_{\mathrm{ld}} + (E - \tilde{E}) - TS + \tfrac{1}{2}\Delta I\omega^2 + \Delta F$$

  where E and S are calculated using the Fermi occupation numbers $f_i = \{1 + \exp[(\epsilon_i - \mu)/T]\}^{-1}$. Here E_{ld} and $\tilde{E}$ are the liquid drop and Strutinsky-averaged energies, respectively.

Static Path Approximation (SPA)

- The *Hubbard-Stratonovich transform* expresses the trace of a two-body interaction as a path integral of a one-body expression:

$$\mathrm{tr}\left[\exp\left(-\frac{\langle\rho, v\rho\rangle}{2T}\right)\right] = \int D[\sigma]\ \exp\left(\frac{\langle\sigma, v\sigma\rangle}{2T}\right)\mathrm{tr}\left[\exp\left(-\frac{\langle\rho, v\sigma\rangle}{T}\right)\right]$$

- Thus the partition function for a two-body Hamiltonian can be written as the functional integral of the partition function for an effective one-body Hamiltonian.

$$\mathcal{Z} = \int D[\sigma]\ \mathcal{Z}_{\mathrm{eff}}[\sigma(t)]$$

- The *Static Path Approximation* consists of reducing the path integral to a simple integration by considering only time-independent paths.

- The *Mean-Field Approximation* consists of choosing the free energy corresponding to the maximum of the SPA integrand.

Thermal Shape Fluctuations

- Large fluctuations around equilibrium shape may exist due to finiteness of nuclear system ($\sim$ 500 d.o.f.s).

- SPA and Gibbs theory gives probability distribution of deformations in a nucleus:

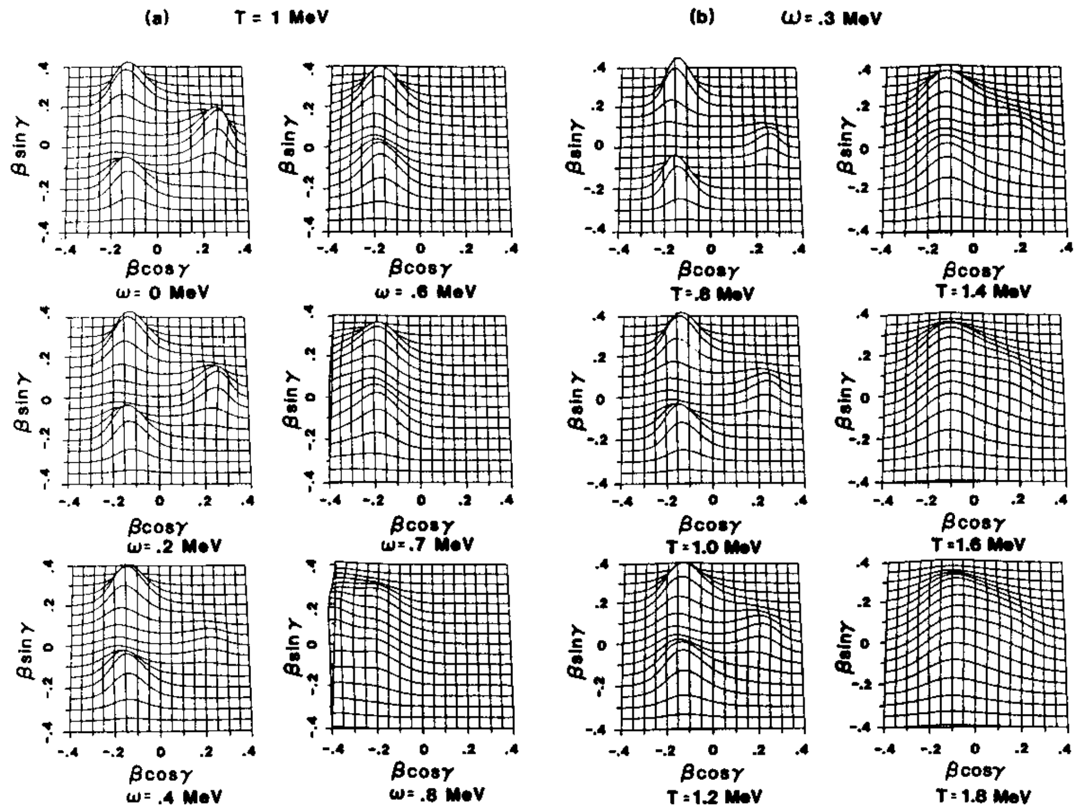
$$P(\alpha_{2\mu}) = \mathcal{Z}^{-1} \exp\left[-F(\alpha)/T\right]$$

  where $F(\alpha) = F(T, \omega_z, \mu_{\rm p}, \mu_{\rm n}; \alpha_{2\mu})$ is the free energy as a function of quadrupole shape $\alpha_{2\mu}$ and

$$\mathcal{Z} = \int d^5\alpha_{2\mu} \exp\left[-F(\alpha)/T\right]$$

  is the SPA for the exact partition function.

- The measure $d^5\alpha_{2\mu} = \beta^4 |\sin 3\gamma| d\beta d\gamma d\Omega$ includes fluctuations in intrinsic shape (β, γ) and in Euler angle $\Omega = (\psi, \theta, \phi)$.



Shape distributions corresponding to the ^{166}Er free energy surfaces shown: (a) at constant temperature T = 1 MeV and (b) at constant angular velocity ω = 0.3 MeV. Notice that the surfaces become more diffuse when either the temperature or angular velocity increases.

Observables

- The value of any adiabatic observable Θ averaged over the shape distribution of the grand canonical ensemble is

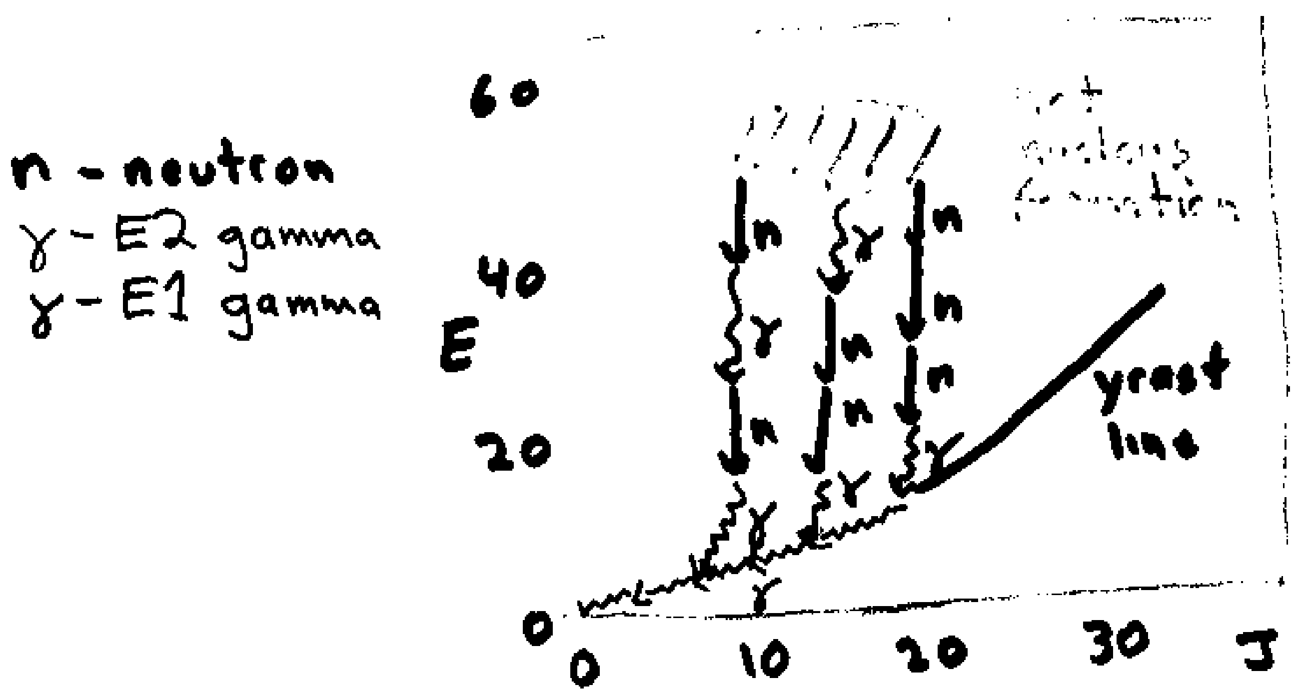
$$\langle\Theta\rangle_{\mathrm{gce}} = \int d^5\alpha_{2\mu} P(\alpha)\Theta(\alpha)$$

- The expectation of Θ over the grand canonical ensemble (T, $\boldsymbol{\omega}$, μ_{p}, μ_{n} representation) can be converted to one over the micro-canonical ensemble (E, $\boldsymbol{J}$, Z, N representation):

$$\langle\Theta\rangle_{\mathrm{mce}} = \frac{1}{32\pi^4}\int_{-i\infty}^{i\infty} d\left(\frac{1}{T}\right)\int_0^{4\pi i} d\left(\frac{\omega_z}{T}\right)\int_0^{2\pi i} d\left(\frac{\mu_{\mathrm{p}}}{T}\right)\int_0^{2\pi i} d\left(\frac{\mu_{\mathrm{n}}}{T}\right) \times \frac{\mathcal{Z}(T,\omega_z,\mu_{\mathrm{p}},\mu_{\mathrm{n}})}{\rho(E,M,Z,N)} e^{(E-\omega_z M-\mu_{\mathrm{p}}Z-\mu_{\mathrm{n}}N)/T}\langle\Theta\rangle_{\mathrm{gce}}$$

Example: Giant Dipole Resonance

Giant Dipole Resonance (GDR): collective oscillation of the proton center-of-mass with respect to the neutron center-of-mass.



GDR Equation of Motion

- Hamiltonian: $H(\mathbf{D},\mathbf{P}) = \frac{1}{2}\mathbf{P}^2 + \frac{1}{2}\mathbf{D}\cdot\mathsf{E}^2\cdot\mathbf{D} - \boldsymbol{\omega}\times(\mathbf{D}\times\mathbf{P})$, where $\mathbf{D}$ is the dipole moment and $\mathbf{P}$ its momentum

- Equations of motion (with damping term):

$$\begin{aligned}\dot{\mathbf{D}} &= \mathbf{P} - \boldsymbol{\omega}\times\mathbf{D} - \tfrac{1}{2}\mathsf{\Gamma}\cdot\mathbf{D}\\ \dot{\mathbf{P}} &= -\mathsf{E}^2\cdot\mathbf{D} - \boldsymbol{\omega}\times\mathbf{P} - \tfrac{1}{2}\mathsf{\Gamma}\cdot\mathbf{P}\end{aligned}$$

  where

$$\begin{aligned}\mathsf{E} &= \text{energy of osc. mode} \propto (\text{axis length})^{-1}\\ \mathsf{\Gamma} &= \text{intrinsic damping width} = \Gamma_0\,(\mathsf{E}/E_0)^{\delta}\end{aligned}$$

- Note that $\mathsf{E} = \mathsf{E}(\alpha_{2\mu})$ and $\mathsf{\Gamma} = \mathsf{\Gamma}(\alpha_{2\mu})$ depend on deformation and are matrices in the laboratory frame.

GDR Cross Section

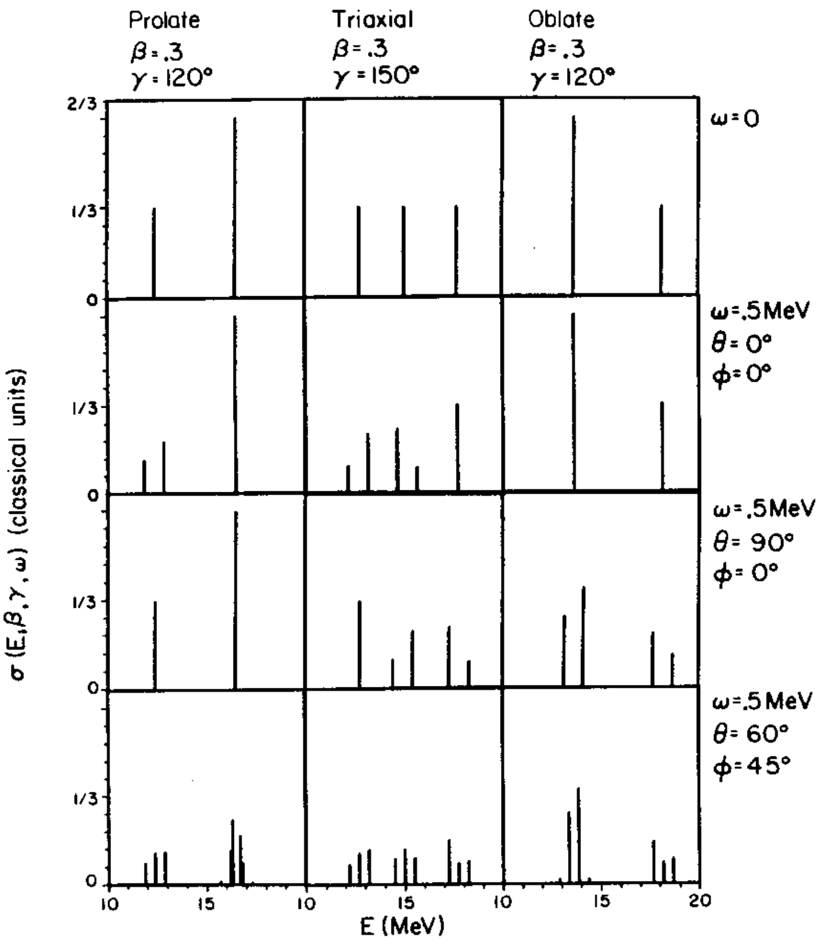
- Absorption cross section is related to Fourier transform of dipole autocorrelation function:

$$\sigma_{\rm abs}(\epsilon) = \frac{2\pi\epsilon}{3c\hbar^2}\int_{-\infty}^{\infty} dt\, e^{i\epsilon t/\hbar} \sum_\mu \langle D_\mu^\dagger(t) D_\mu(0)\rangle$$

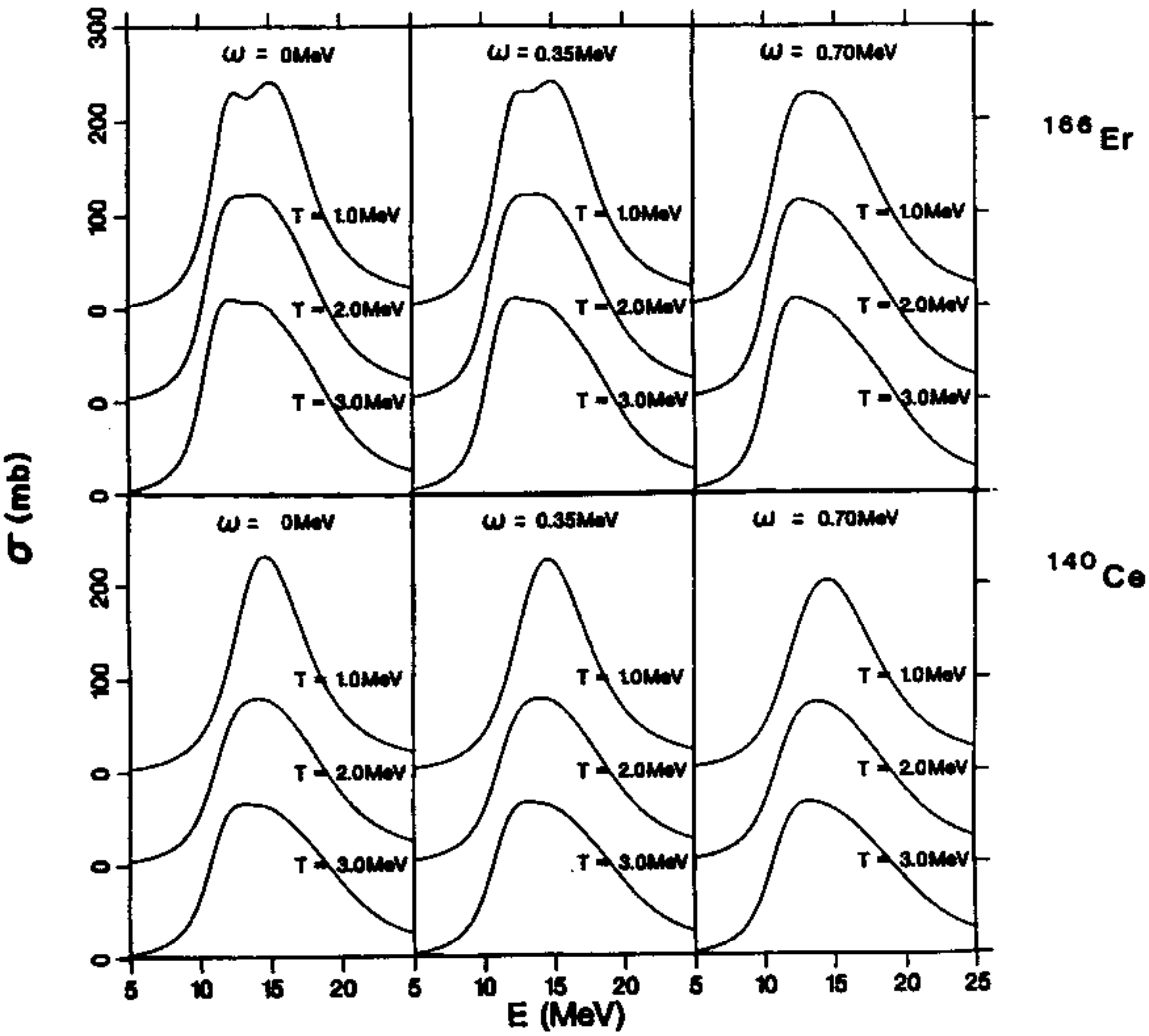
- Solution to e.o.m. shows the Fourier transform of $\sum_\mu D_\mu^\dagger(t)D_\mu(0)$ is the sum of nine Lorentzian shapes:

$$f_k = \Gamma_k \epsilon^2 \Big/ \left[\left(\epsilon^2 - E_k^2\right)^2 + \Gamma_k^2\epsilon^2\right]$$

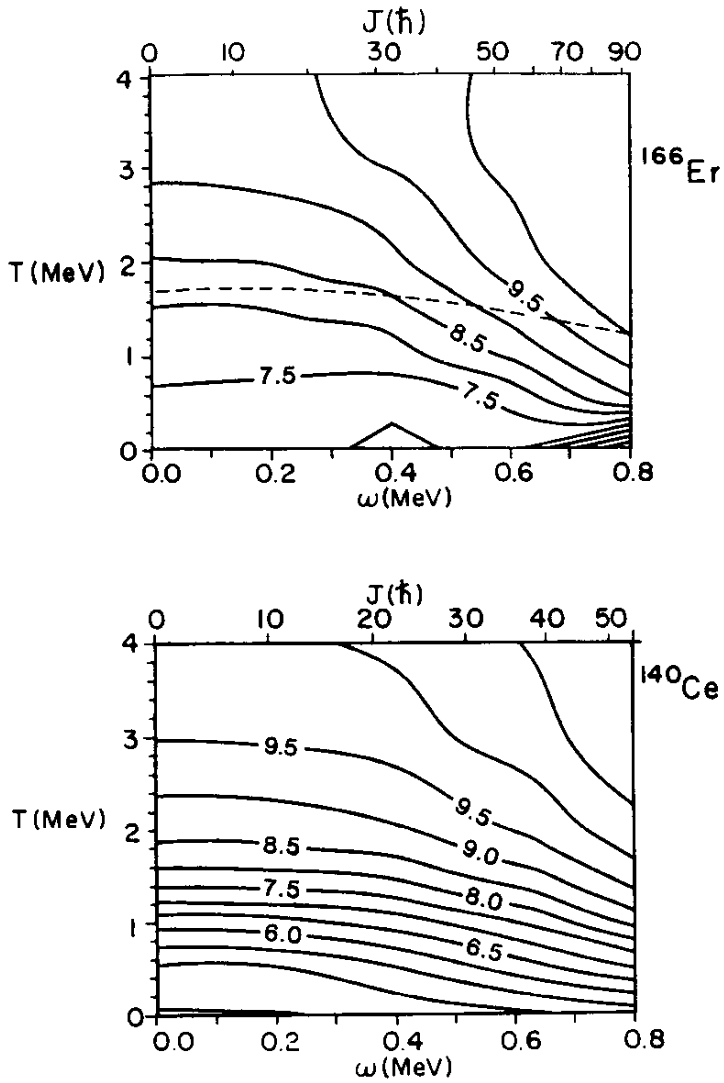
- The average $\langle\cdot\rangle$ is performed over all shapes $\alpha_{2\mu}$.
- Features:
  - includes shape fluctuations
  - only three free parameters (E_0, Γ_0, δ), determined from ground-state GDR properties



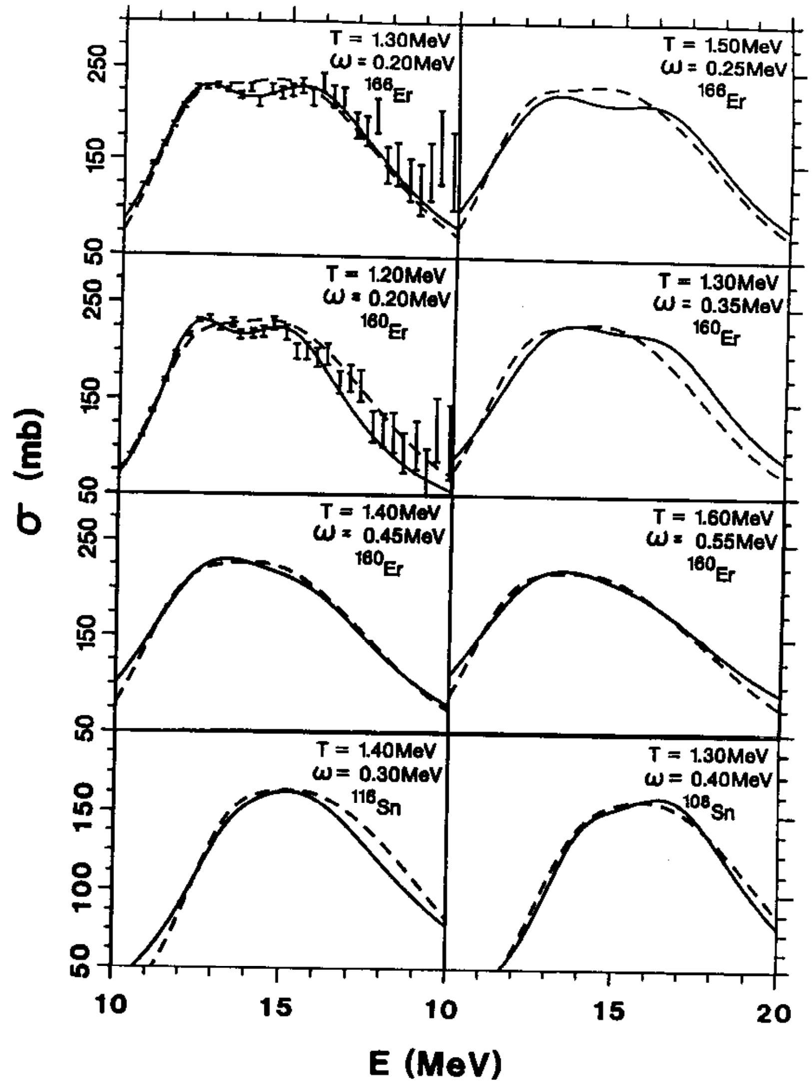
The splitting of the various GDR components due to deformation and rotation: examples of $\sigma(\epsilon;\beta,\gamma,\Omega)$ with no spreading for several typical intrinsic deformations (β,γ), orientations Ω with respect to the rotation axis, and angular velocities ω. We have set $\Omega = (0,\theta,\phi)$ and $\omega = (0,0,\omega)$ without loss of generality, since specifying both the Euler angles and the orientation of the angular velocity is redundant. In the plots $E_0 = 15$ MeV and $\Gamma_0 = 0$.



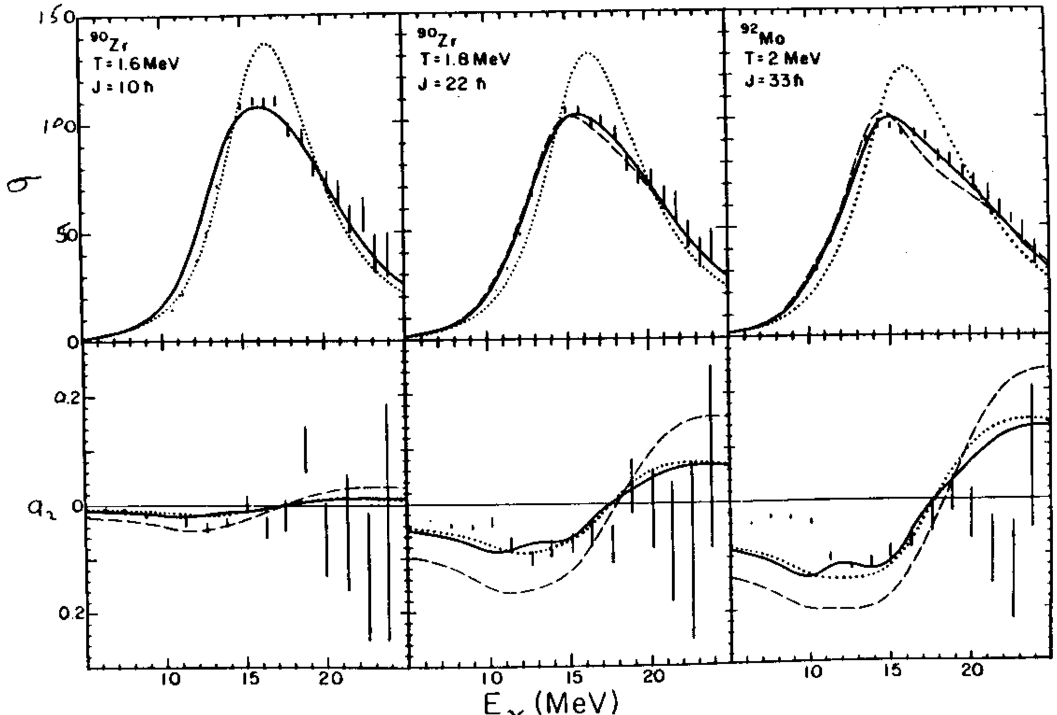
Calculated GDR absorption cross-sections at various temperatures and angular velocities. Top: for ^{166}Er; bottom: for ^{140}Ce. The only free parameters, E_0, Γ_0, and δ, were determined from the ground state GDR.



FWHM contour lines (MeV) of GDR cross-section in intensive (T,ω) variables for ^{166}Er (top) and ^{140}Ce (bottom). The dashed line in the upper figure is the transition line separating the triaxial and oblate phases.



Comparison of calculated and experimental ^{160,166}Er and ^{108,116}Sn GDR cross-sections: the solid lines show the Lorentzians from CASCADE fits to the experimental cross-section and the dashed ones show our calculations. Note that the error bars are only suggestive the the experimental accuracy.



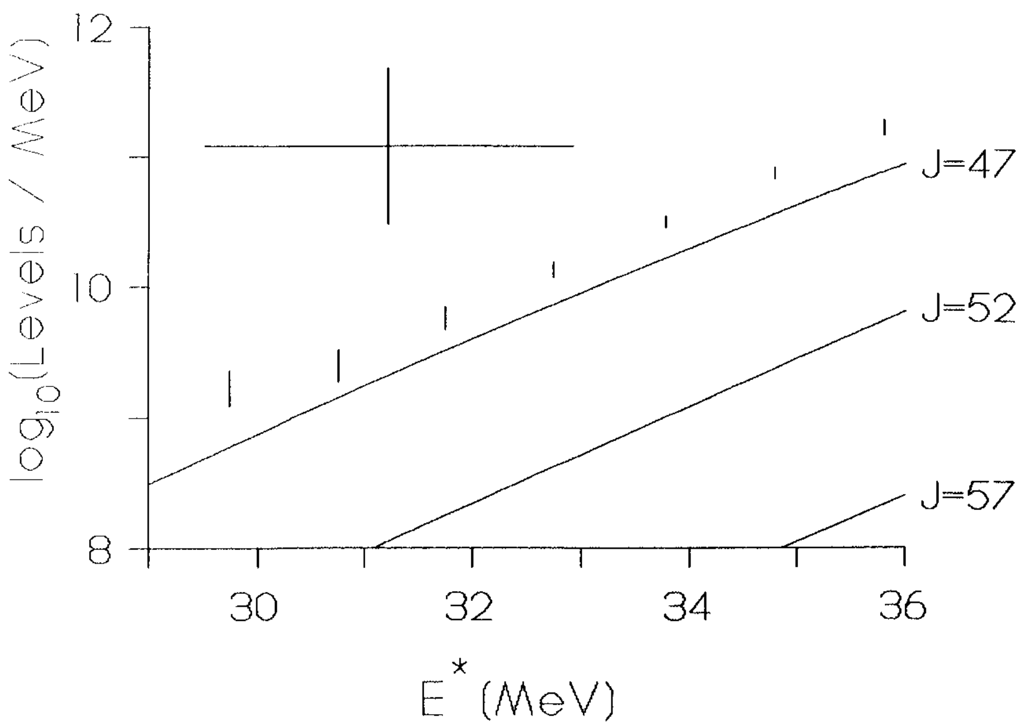
Comparison between theory and experiment for $\sigma(\varepsilon)$ and $a_2(\varepsilon)$ in excited ^{90}Zr and ^{92}Mo in the ature range $T \approx 1.6$–2 MeV and spins range $J \approx 10$–$33\hbar$. The experimental data [17]) is shown by or bars. The calculations of the full fluctuation theory with the unitary metric (3.3) including ation fluctuations are the solid lines. The dashed lines show the results of average over (β, γ) but t orientation fluctuations. The dotted lines are the results of similar calculations but with the metric (3.18) and without orientation fluctuations.

Example: Level Density

- SPA Level density involves integration over shape distribution and inversion of Laplace transforms:

$$\rho(E, M, Z, N) = \int \frac{d^5\alpha_{2\mu}}{32\pi^4} \int_{-i\infty}^{i\infty} d\left(\frac{1}{T}\right) \int_0^{4\pi i} d\left(\frac{\omega_z}{T}\right) \int_0^{2\pi i} d\left(\frac{\mu_\mathrm{p}}{T}\right) \int_0^{2\pi i} d\left(\frac{\mu_\mathrm{n}}{T}\right)$$
$$\times\, e^{(E-\omega_z M-\mu_\mathrm{p} Z-\mu_\mathrm{n} N)/T} \exp[-F(T, \omega_z, \mu_\mathrm{p}, \mu_\mathrm{n}; \alpha)/T]$$

- Evaluation of the 9 integrals:
  - 1 done analytically
  - 3 via analytic application of method of steepest descents (saddle-point method)
  - 1 via numeric application of method of steepest descents
  - 4 done numerically using Gaussian integration



Comparison of SPA level density at three spins ($J = 47, 52, 57\ \hbar$) to experimental data for ^{166}Er. The large cross in the upper-left of the figure indicates the experiment's systematics errors in level density and energy. The spin in the experiment is $52 \pm 5\ \hbar$.

Time-Dependent Fluctuations

- If the shape fluctuates too rapidly, observables (such as $\mathbf{D}$) do not have time to respond to the shape changes.
- This nonadiabaticity results in “motional narrowing” of cross sections and could explain overestimates of GDR width by the adiabatic theory at high temperature ($T \geq 2$ MeV).
- *Stochastic Approach:* use a Brownian motion model (Langevin Equation) to describe $\alpha_{2\mu}(t)$ trajectories.

Stochastic Shape Changes (I)

- Langevin equation:

$$\underbrace{B\ddot{\alpha}_{2\mu}}_{\text{inertia}} + \underbrace{\chi\dot{\alpha}_{2\mu}}_{\text{damping}} + \underbrace{\frac{\partial F}{\partial \alpha^*_{2\mu}}}_{\text{external force}} = \underbrace{\chi f_{2\mu}(t)}_{\text{random force}}$$

- The random force is a Gaussian random process:

$$\langle f_{2\mu} \rangle = 0, \qquad \langle f_{2\mu}(s) f^*_{2\mu}(t) \rangle = \xi \, \delta_{\mu\nu} \delta(s-t)$$

- Fluctuation-dissipation theorem: $\xi = 2T/\chi$.

Stochastic Shape Changes (II)

- Overdamped case:

$$\dot{\alpha}_{2\mu} = -\frac{1}{\chi}\frac{\partial F}{\partial \alpha^*_{2\mu}} + f_{2\mu}(t) \quad \text{for} \quad \left(\frac{\chi}{B}\right)^2 \gg \frac{1}{B}\max\left\{\left|\frac{\partial^2 F}{\partial \alpha_{2\mu}\partial \alpha_{2\nu}}\right|\right\}$$

- Fokker-Planck equation for probability $P(\alpha, t)$:

$$\frac{\partial P}{\partial \alpha_{2\mu}} = \frac{\partial}{\partial \alpha_{2\mu}}\left(\frac{1}{\chi}\frac{\partial F}{\partial \alpha^*_{2\mu}}P\right) + \tfrac{1}{2}\xi^2\frac{\partial^2 P}{\partial \alpha_{2\mu}\partial \alpha^*_{2\mu}}$$

Stochastic Shape Changes (III)

- Stationary solution / equilibrium distribution:

$$\frac{d}{dt}P(\alpha,t)=0$$

implies

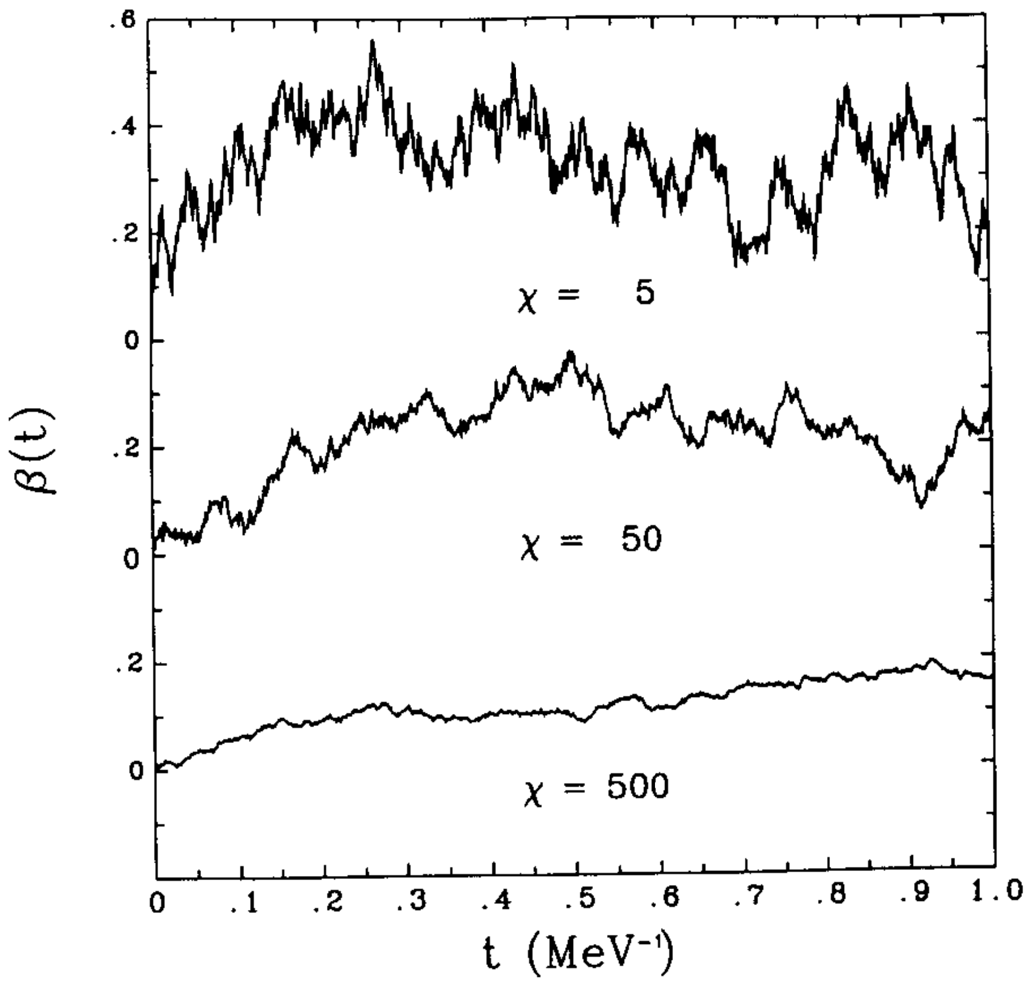
$$P(\alpha,t)=P_{\mathrm{eq}}(\alpha)=\exp[-F(\alpha)/T]$$

- Convergence to equilibrium:

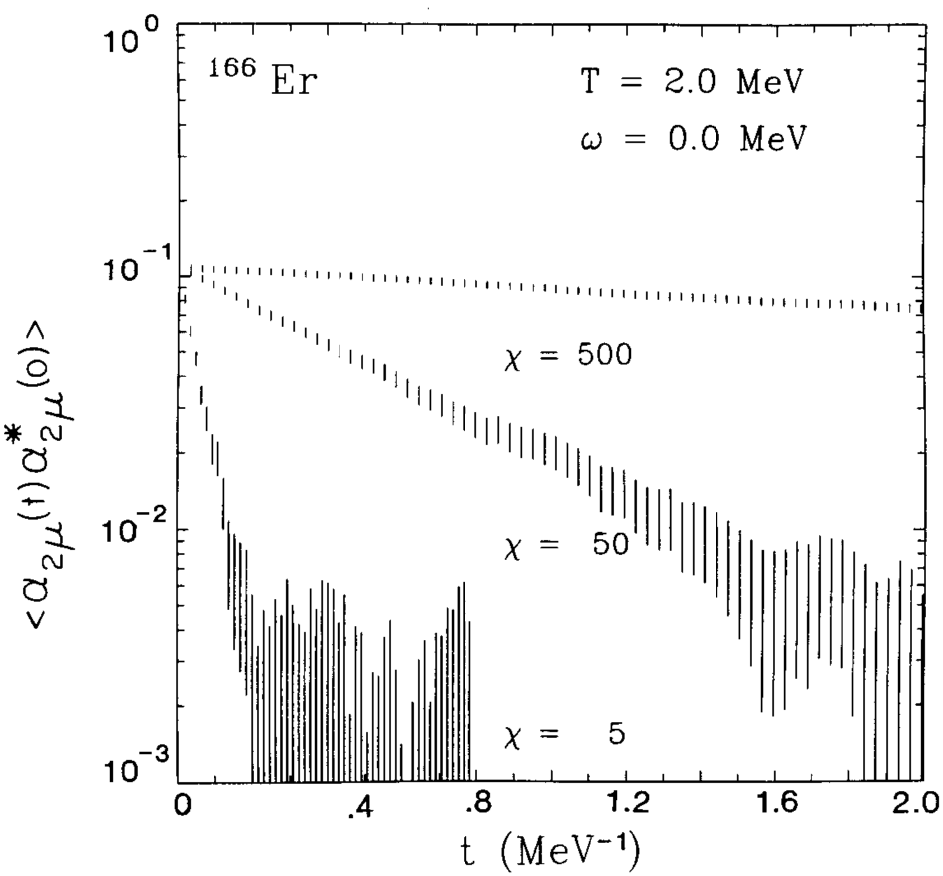
$$\frac{d}{dt}\int d^5\alpha_{2\mu}\,[-P(\alpha)\ln P(\alpha)]\geq 0$$

so

$$P(\alpha,t\to\infty)=P_{\mathrm{eq}}(\alpha)$$



Typical $\beta(t)$ trajectories for $\chi = 5$, 50 and 500 (from top to bottom) obtained by a Monte-Carlo solution. The nucleus is ^{166}Er at $T = 2$ MeV and $\omega = 0$. Notice that as the process becomes more sudden (χ gets smaller) the changes in β are more erratic.



The correlation function $<\alpha_{2\mu}(t)\alpha_{2\mu}{}^*(0)>$ (on a logarithmic scale) versus t. Notice that the decay is approximately exponential. The bars denote the statistical Monte-Carlo errors

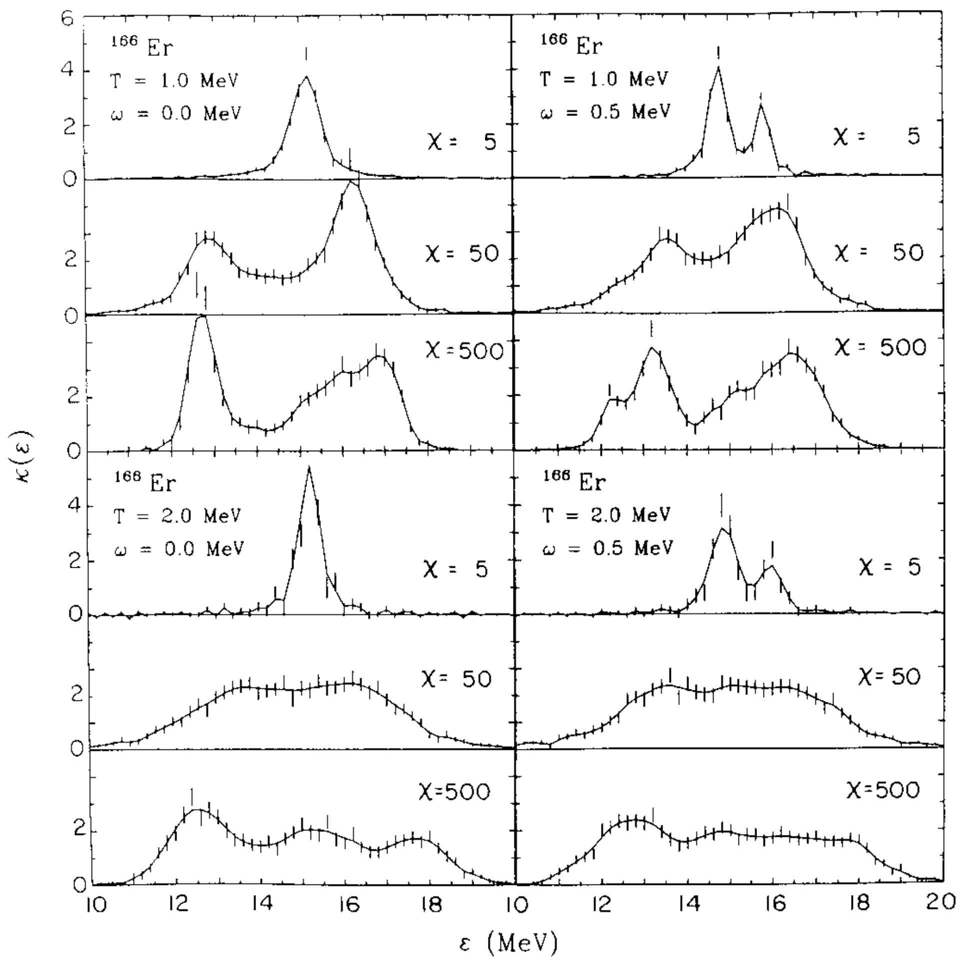
GDR with Dynamically Changing Shape

The equations of motion are the same as in the adiabatic case, except they must be solved numerically since $\mathbf{E}(\alpha_{2\mu})$ and $\mathbf{\Gamma}(\alpha_{2\mu})$ are now time-dependent random variables.

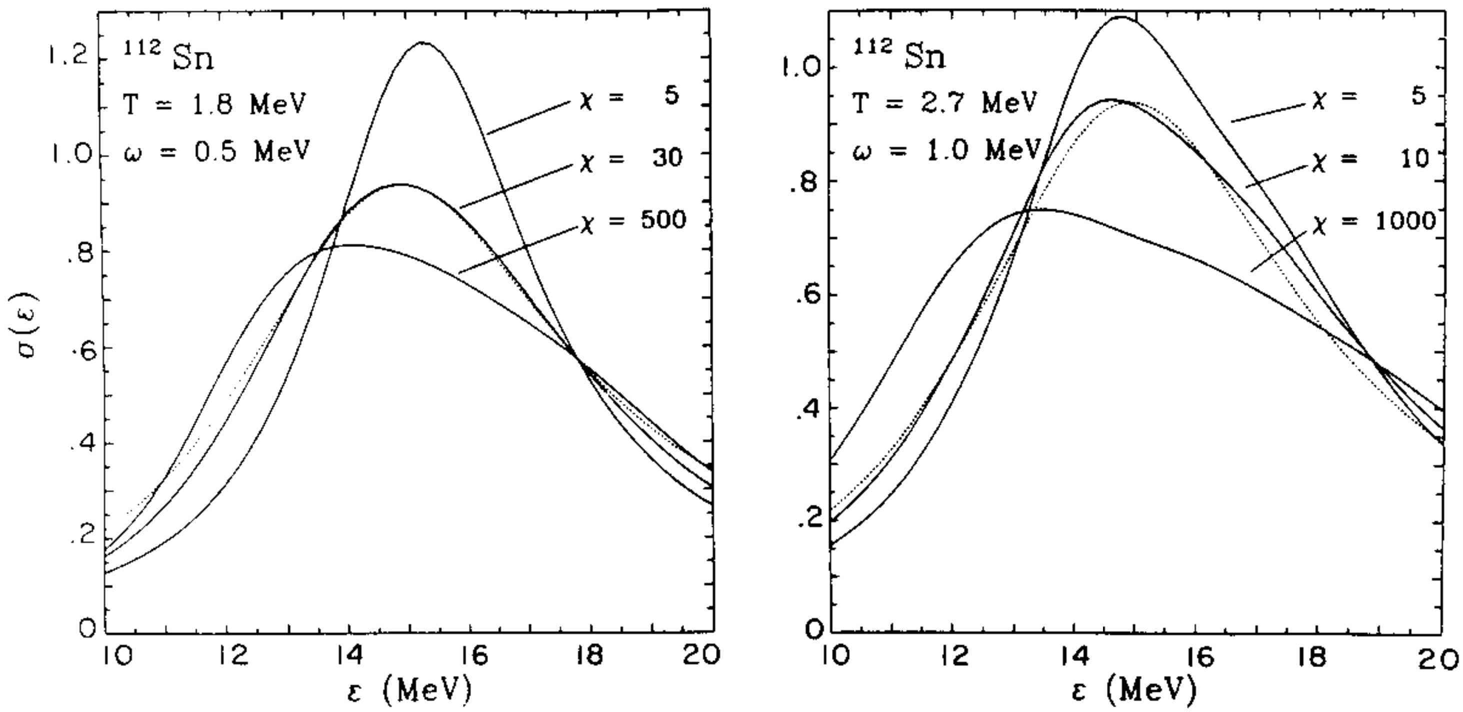
We use a stochastic version of the Runge-Kutta method to solve the equations numerically. Several hundred trajectories must be considered to get statistically significant results.

Once again, the Fourier transform of the dipole autocorrelation function $\sum_{\mu}\langle D_{\mu}^{\dagger}(t)D_{\mu}(0)\rangle$ is the absorption cross section.

The new parameter χ determines the degree of adiabaticity.



Systematics of the Fourier transform $\kappa(\epsilon)$ of the dipole correlation function for a typical deformed nucleus ^{166}Er using the $\Gamma = 0$ model. We show for several T and ω the function $\kappa(\epsilon)$ in the adiabatic ($\chi = 500$), intermediate ($\chi = 50$) and sudden ($\chi = 5$) cases. The bars denote the statistical Monte-Carlo errors; the curves are interpolate the results.



Comparison with experimental GDR cross-sections for ^{112}Sn. The dotted lines show the experimental cross-sections. The solid lines are the results of our stochastic model for several values of χ. The higher and lower values of χ are the adiabatic and sudden limits, respectively. The intermediate values of χ show the best fit to the data.

Shape Transitions in the Shell Model

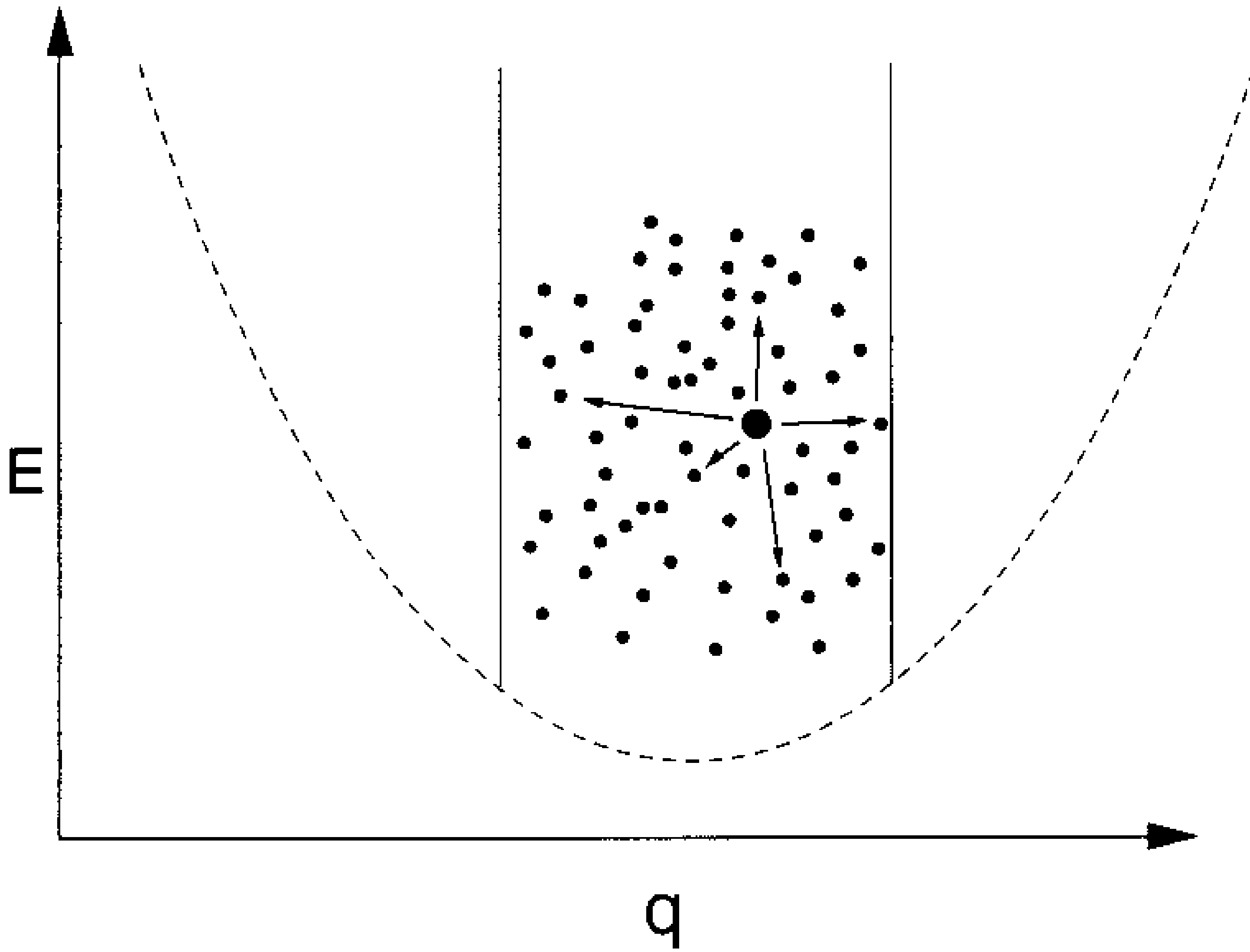
- *Basic assumption:* The highly excited nucleus can be described as an incoherent mixture of Hartree-Fock (HF) configurations at a given energy.

- Since we assume a basis of static HF solutions, the dynamics comes entirely from the residual interaction. For high enough level density, the rate of depopulation for a state i may be calculated from Fermi's Golden Rule:

$$\Gamma = \frac{2\pi}{\hbar} \sum_f \left| \langle i | v_{\text{residual}} | f \rangle \right|^2 \delta(E_f - E_i)$$

  where the sum is over HF states f differing by the orbital assignment of two particles.

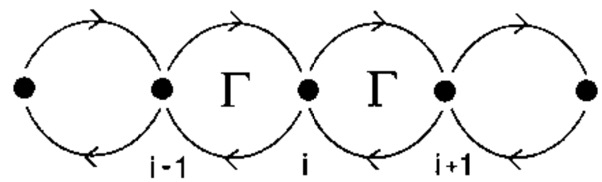
- Because each HF state has its own shape, the nuclear shape will change with each transition. However, the final states f will not differ greatly in deformation from the initial state i because only two particles have changed orbit.


E
q

Diffusion Model

- We discretize deformations β_i and construct a rate equation for populations $P_i = P(\beta_i, t)$:

$$\frac{dP_i}{dt} = \Gamma P_{i-1} - 2\Gamma P_i + \Gamma P_{i+1} \approx \Gamma(\Delta\beta)^2 \frac{\partial^2 P}{\partial \beta^2}$$



- In the continuous limit we have the classical diffusion equation:

$$\frac{\partial P}{\partial t} = D_\beta \frac{\partial^2 P}{\partial \beta^2}$$

with diffusion coefficient D_β given by the transition rate weighted by the square of the jump in deformation:

$$D_\beta = \frac{2\pi}{\hbar} \sum_f (\beta_i - \beta_f)^2 \left| \langle i | v_{\mathrm{residual}} | f \rangle \right|^2 \delta(E_f - E_i)$$

Simple Estimate of D_β (I)

To estimate D_β we separately calculate the lifetime of the HF configurations and the mean-square change in quadrupole deformation when two particles change their orbits.

- mean-square change in deformation:
  - The mean-square dispersion in harmonic oscillator shell with n quanta of excitation is

$$\langle {Q_{\mathrm{sp}}}^2 \rangle = \left(\frac{\hbar}{m\tilde{\omega}} \right)^2 \frac{n(n+3)}{2} \, .$$

  - Core polarization increases the quadrupole moment by approximately a factor of two.
  - The quadrupole moment and the deformation are related by

$$Q = 6\sqrt{\frac{5}{4\pi}} A {R_0}^2 \beta$$

    for near-spherical shapes.

Simple Estimate of D_β (II)

- Therefore the β dispersion is

$$\langle(\beta_i - \beta_f)^2\rangle \approx 6.8n(n+3)A^{-8/3} \approx 9.0/A^2 \; .$$

- lifetime of HF configurations:
  - From particle transfer reactions we have empirical information about lifetimes of single-particle states at moderate excitation:

$$\Gamma_{\rm sp}(E_{\rm ex}) = {E_{\rm ex}}^2/(20 \text{ MeV}) \; .$$

    The amount of phase space available at temperature T is related to an excitation energy above a cold Fermi sphere by $E_{\rm ex} = \pi T$.
  - The average number of particles is determined by integrating the Fermi function:

$$N_{\rm p} = \int_0^\infty dn[e^{(\epsilon - \epsilon_{\rm f})/T} + 1]^{-1} \approx \ln 2 \frac{dn}{d\epsilon} T$$

    so

$$N_{\rm p} + N_{\rm h} \approx \tfrac{3}{2} \ln 2 \, AT/\epsilon_{\rm f}$$

    since $dn/d\epsilon \approx 3A/4\epsilon_{\rm f}$.

Simple Estimate of D_β (III)

– The thermal width is given by the single particle width times the average number of particles and holes:

$$\Gamma_{\rm thermal} \approx AT^3/\epsilon_{\rm f}/(1.9\ {\rm MeV}) \approx AT^3/(61\ {\rm MeV}^2)$$

for $\epsilon_{\rm f} = 32\ {\rm MeV}$.

- diffusion coefficient:

$$D_\beta \approx \langle(\beta_i - \beta_f)^2\rangle\, \Gamma_{\rm thermal} \approx T^3/A(6.9\ {\rm MeV}^2)$$

- example: for ^{76}Ge at $T = 2.5\ {\rm MeV}$, $D_\beta = 28\ {\rm keV}$.

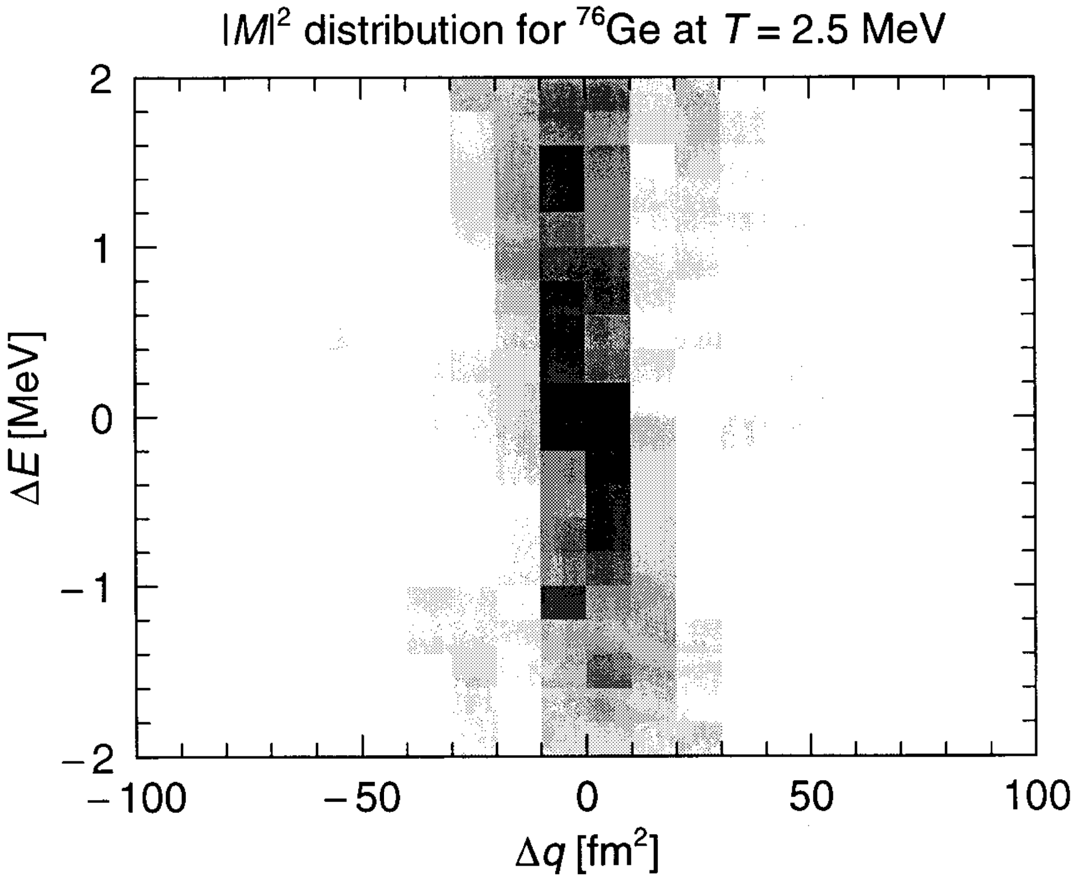
Microscopic Calculation of the Diffusion Coefficient

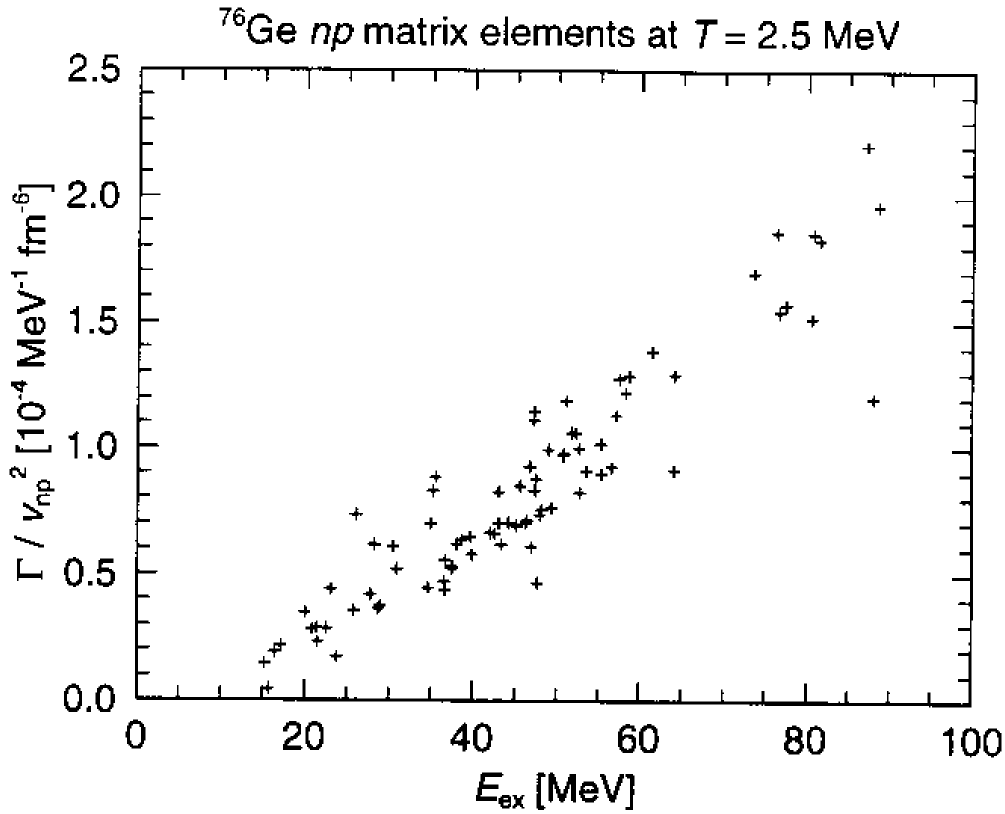
We calculate the $\langle i|v_{\mathrm{residual}}|f\rangle$ matrix elements in

$$D_Q = \frac{2\pi}{\hbar}\sum_f (Q_i - Q_f)^2 \left|\langle i|v_{\mathrm{residual}}|f\rangle\right|^2 \delta(E_f - E_i)$$

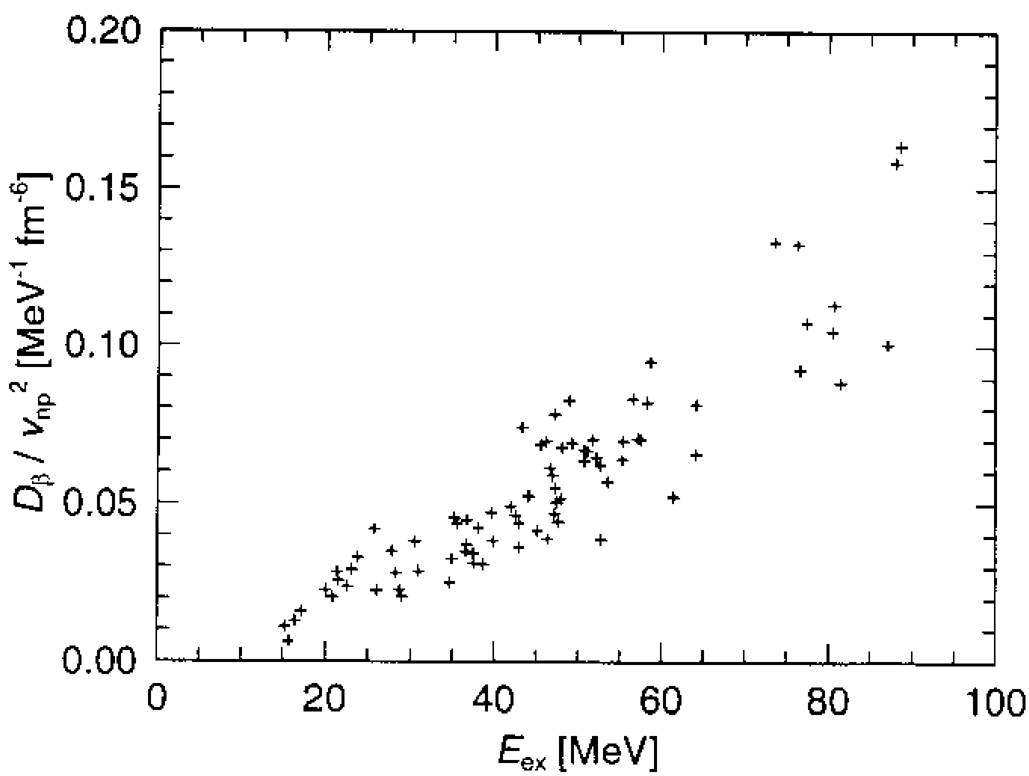
for a Serber-force δ-function residual interaction using Nilsson model wavefunctions and energy levels. The sum above is estimated using Monte-Carlo methods.

The strength of the pp and nn residual interaction is determined from a G-matrix parameterization appropriate for valence particles. The np strength comes from an empirical interaction extracted from the spectra of odd-odd nuclei.

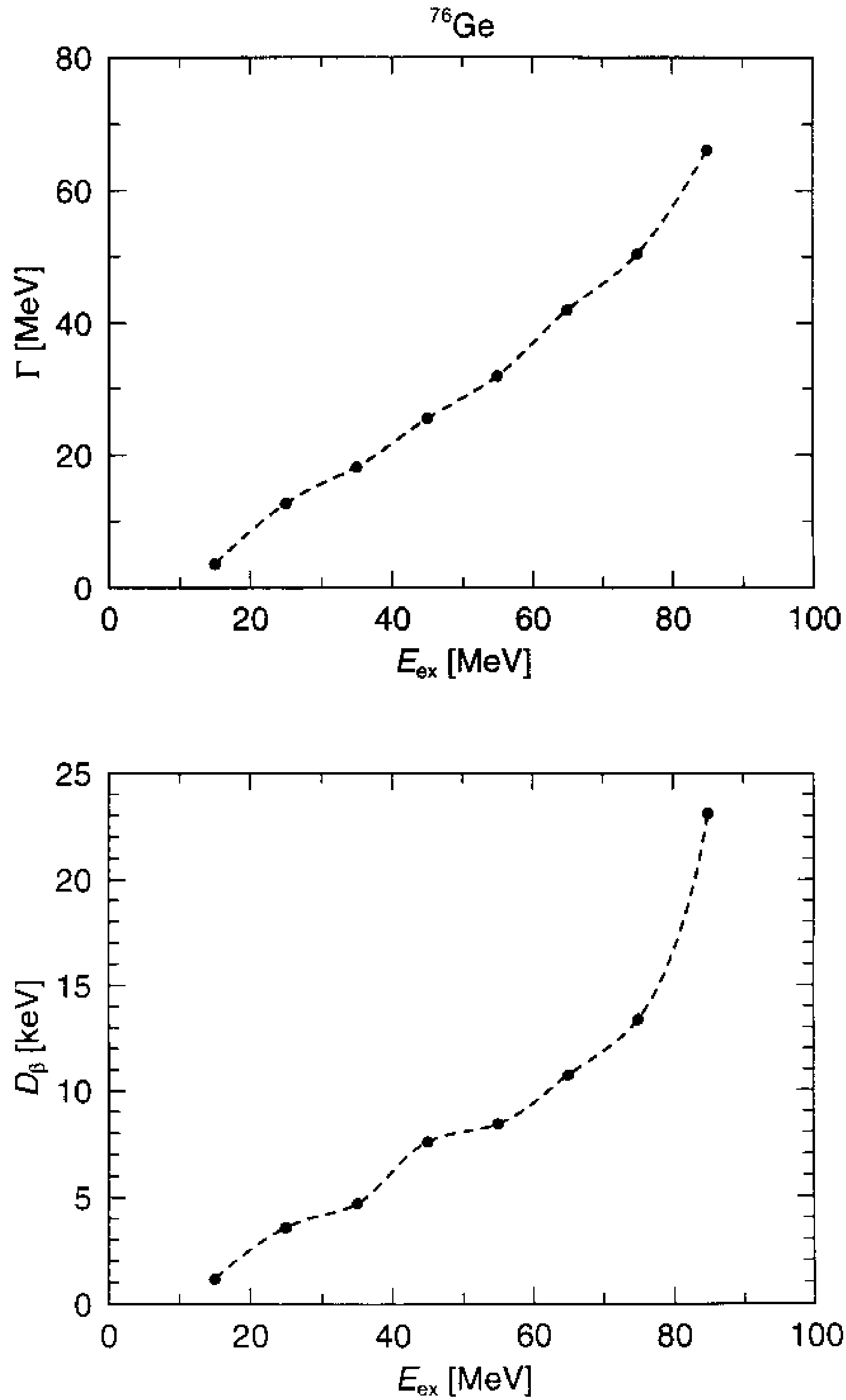

|M|² distribution for ⁷⁶Ge at T = 2.5 MeV
2
1
0
−1
−2
ΔE [MeV]
−100
−50
0
50
100
Δq [fm²]



^{76}Ge np matrix elements at $T = 2.5$ MeV
Γ / v_{np}^2 [10^{-4} MeV^{-1} fm^{-6}]
E_{ex} [MeV]
0.0
0.5
1.0
1.5
2.0
2.5
0
20
40
60
80
100



D_β / v_{np}^2 [MeV^{-1} fm^{-6}]
E_{ex} [MeV]
0.00
0.05
0.10
0.15
0.20
0
20
40
60
80
100



76Ge
80
60
40
20
0
Γ [MeV]
0
20
40
60
80
100
E_{ex} [MeV]
25
20
15
10
5
0
D_β [keV]
0
20
40
60
80
100
E_{ex} [MeV]

Calculated Diffusion Coefficients

(T = 2.5 MeV)

Nucleus	Γ (MeV)	D_β (keV)	D_β/Γ (10^{-3})
^{24}Mg	3.10	30.	10.0
^{76}Ge	26.3	7.3	0.28
^{110}Sn	29.5	4.6	0.16
^{158}Er	56.5	3.8	0.067

Other Theoretical Estimates of D_β

- *Diabatic Friction* (Nörenberg, 1981): Dissipation arises from the diabatic production of particle-hole excitations which are subsequently equilibrated by two-body collisions. This yields the same functional dependence of A and T as we have and gives numerically similar results.

- *Linear Response* (Yamaji *et al.*, 1988): Inertia, friction, and stiffness coefficients are calculated within a locally harmonic approximation in linear response theory.

- *Memory-Dependent Transport* (Ayik *et al.*, 1991): Memory effects are added to a Boltzmann transport equation. The diffusion coefficient is expressed as a collision integral.

Experimental Measurement of D_β

- *Prescission neutrons:* The time to reach the fission saddle point is directly related to the diffusion coefficient. Measurements of pre-fission neutron multiplicities can be compared to statistical model calculations to infer the diffusion coefficient.

- *Dipole Narrowing:* The reduction of the GDR cross-section width due to motional narrowing can be used to estimate the adiabaticity parameter χ, which is related to the diffusion coefficient by $D_\beta = T/\chi$.

Comparison of Theory and Experiment

(A = 158, T = 2 MeV)

Source	D_β	Reference
Theory:		
Present calculation	2.0 keV	Bush *et al.* (1992)
Diabatic friction	2.7 keV	Nörenberg (1981)
Linear response	12.4 keV	Yamaji *et al.* (1988)
Experiment:		
Prescission neutrons	$\geq$ 40 keV	Gavron *et al.* (1987)
Dipole narrowing	50 keV	Alhassid *et al.* (1990)

Conclusions and Prospects

- Shape fluctuations must be considered when studying hot rotating nuclei.
- The giant dipole resonance is observably affected by shape fluctuations. The fluctuation theory provides good predictions of the experimental GDR cross section.
- Current theory predicts a nucleus which is more dissipative than available experiments seem to indicate.
  - Make improved theoretical estimates.
  - Reanalyze experiments to better extract D_β and seek more experiments from which D_β can be extracted.
- Work is now underway to account for the effect of time-dependent shape changes upon level densities in hot rotating nuclei.